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Abstract

We combine a Korn type inequality with Widman's hole filling technique to prove the interior regularity of minimizers for energies occurring in General Relativity. In addition we provide a new variant of this Korn type inequality valid for the nonquadratic case.

In a recent paper Dain [Da] proves Korn type inequalities in which the symmetric gradient

$$\varepsilon(u) = \frac{1}{2}\left(\partial_i u^j + \partial_j u^i\right)_{1\le i,j\le n}$$

of vector fields $u : \Omega \to \mathbb{R}^n$ defined on a bounded Lipschitz domain $\Omega \subset \mathbb{R}^n$ is replaced by its trace free part

$$\varepsilon^D(u) = \varepsilon(u) - \frac{1}{n}(\operatorname{div} u)\,\mathbf{1},$$

where $\mathbf{1}$ denotes the unit-matrix. More precisely, it is shown that in case $n \ge 3$ it holds

$$\|u\|^2_{W^1_2(\Omega)} \le C\left[\int_\Omega |u|^2\,dx + \int_\Omega \left|\varepsilon^D(u)\right|^2\,dx\right] \tag{1}$$

for all functions u from the Sobolev space $W^1_2(\Omega;\mathbb{R}^n)$ (see [Ad] for a definition), whereas for $n = 2$ we have

$$\int_\Omega |\nabla\varphi|^2\,dx \le 2\int_\Omega \left|\varepsilon^D(\varphi)\right|^2\,dx \tag{2}$$

valid now for fields $\varphi \in C^\infty_0(\Omega;\mathbb{R}^2)$. By approximation (2) extends to the space $\overset{\circ}{W}{}^1_2(\Omega;\mathbb{R}^2)$ of Sobolev functions with zero trace, and another application of (2) yields

$$X := \left\{u \in L^2_{\mathrm{loc}}(\Omega;\mathbb{R}^2) : \varepsilon^D(u) \in L^2_{\mathrm{loc}}(\Omega;\mathbb{S}^2)\right\} = W^1_{2,\mathrm{loc}}(\Omega;\mathbb{R}^2). \tag{3}$$

(Here $\mathbb{S}^2$ denotes the space of symmetric (2×2)-matrices.) In fact, if $w \in X$, then we consider $\eta\, w^{(\rho_\nu)}$, where η is a localization function and $w^{(\rho_\nu)}$ is a sequence of mollifications. Since (2) is true for $\eta\, w^{(\rho_\nu)}$, we easily obtain that w belongs to $W^1_{2,\mathrm{loc}}(\Omega;\mathbb{R}^2)$. The need for inequalities of the form (1) or (2) origins from the question, if functionals of the form

$$E(u,\Omega) := \int_\Omega \left(\left|\varepsilon^D(u)\right|^2 - g\cdot u\right)\,dx \tag{4}$$

studied in General Relativity (see [BI]) are coercive under suitable boundary conditions. Due to (1) and (2) the answer is positive, and the goal of our paper is the investigation of the smoothness properties of minimizers of functionals being more general than E introduced in (4) at least for the $2D$–case. To be precise, consider an energy density $f : \mathbb{S}^2 \to [0, \infty)$ of class C^2 such that with constants $\lambda, \Lambda > 0$ it holds

$$\lambda\,|\sigma|^2 \le D^2 f(\tau)(\sigma,\sigma) \le \Lambda\,|\sigma|^2 \tag{5}$$

for all $\sigma, \tau \in \mathbb{S}^2$. Suppose further that Ω is a domain in $\mathbb{R}^2$ and that we are given functions

$$u_0 \in W^1_2(\Omega;\mathbb{R}^2),\ \ g \in L^\infty(\Omega;\mathbb{R}^2)\,. \tag{6}$$

We then define the energy

$$I[u,\Omega] := \int_\Omega \left[f(\varepsilon^D(u)) - g\cdot u\right]\,dx \tag{7}$$

on the class

$$\mathbb{K} := u_0 + \overset{\circ}{W}{}^1_2(\Omega;\mathbb{R}^2) \tag{8}$$

and get

THEOREM 1. *Let (5) - (8) hold. Then we have:*

a) The problem $I[\cdot,\Omega] \to \min$ *on* $\mathbb{K}$ *admits a unique solution* $u \in \mathbb{K}$.

b) This solution belongs to the space $C^{1,\alpha}(\Omega;\mathbb{R}^2)$ *for some* $\alpha \in (0,1)$.

From the proof we will deduce

Corollary 1. *Let (5) hold and consider* $u \in X$*, which locally minimizes the functional* I *from (7) with* $g \in L^\infty_{\mathrm{loc}}(\Omega;\mathbb{R}^2)$*, i.e.* $I[u,\Omega'] < \infty$ *and* $I[u,\Omega'] \le I[v,\Omega']$ *for all* $v \in X$ *such that* $\mathrm{spt}(u-v) \Subset \Omega'$*, where* Ω' *is any subdomain of* Ω *such that* $\overline{\Omega'}$ *is a compact subset of* Ω*. Then the first derivatives of* u *are locally Hölder continuous in* Ω.

Proof of Theorem 1

Step 1: Existence and uniqueness of the minimizer

For any function $v \in \mathbb{K}$ it holds (on account of (5))

$$
\begin{aligned}
\int_\Omega f\left(\varepsilon^D(v)\right)\,dx \;&\geq\; c\left[\int_\Omega \left|\varepsilon^D(v)\right|^2\,dx - |\Omega|\right]\\
&\geq\; c\left[\int_\Omega \left|\varepsilon^D(v-u_0)\right|^2\,dx - \int_\Omega |\nabla u_0|^2\,dx - |\Omega|\right]\\
&\overset{(2)}{\geq}\; c\left[\int_\Omega |\nabla(v-u_0)|^2\,dx - \int_\Omega |\nabla u_0|^2\,dx - |\Omega|\right]\\
&\geq\; c\left[\int_\Omega |\nabla v|^2\,dx - \int_\Omega |\nabla u_0|^2\,dx - |\Omega|\right]\,,
\end{aligned}
$$

where here and in what follows c represents a positive constant being independent of v, whose value may change from line to line. With Poincaré's and Young's inequality we obtain for any $\delta > 0$

$$
\begin{aligned}
\left|\int_\Omega g\cdot v\,dx\right| \;&\leq\; \delta\int_\Omega |v-u_0|^2\,dx + c(\delta)\int_\Omega |g|^2\,dx + c\int_\Omega |g||u_0|\,dx\\
&\leq\; \delta c\int_\Omega |\nabla(v-u_0)|^2\,dx + c(\delta)\int_\Omega |g|^2\,dx + c\int_\Omega |u_0||g|\,dx\,,
\end{aligned}
$$

so that appropriate choice of δ implies

$$
I[v,\Omega] \geq c\int_\Omega |\nabla v|^2\,dx - c\left(|\Omega|,\lambda,\Lambda,\|g\|_{L^2},\|u_0\|_{W^1_2}\right)\,, \tag{9}
$$

and alternatively we can replace $\int_\Omega |\nabla v|^2\,dx$ on the r.h.s. by $\int_\Omega |\nabla v - \nabla u_0|^2\,dx$. If $\{u_n\}$ denotes on I-minimizing sequence in $\mathbb{K}$, then (9) clearly implies

$$
\sup_n \|u_n\|_{W^1_2(\Omega)} < \infty\,,
$$

so that after passing to a subsequence we can assume $u_n \rightharpoonup: u$ in $W^1_2(\Omega;\mathbb{R}^2)$, $u_n \to u$ in $L^2(\Omega;\mathbb{R}^2)$ for a function $u \in \mathbb{K}$. The lower semicontinuity of $I[\cdot,\Omega]$ yields that u is a solution of the minimization problem. Suppose that $\tilde{u} \in \mathbb{K}$ is a second solution. If $\varepsilon^D(u) \neq \varepsilon^D(\tilde{u})$ holds on a set of positive measure, then the strict inequality

$$
f\left(\varepsilon^D\left(\frac{u+\tilde{u}}{2}\right)\right) < \frac{1}{2}f\left(\varepsilon^D(u)\right) + \frac{1}{2}f\left(\varepsilon^D(\tilde{u})\right)
$$

is true on this set which leads to the contradiction

$$I\left[\frac{u+\tilde{u}}{2},\Omega\right] < \inf_{\mathbb{K}} I\,.$$

We therefore have $\varepsilon^D(u-\tilde{u}) = 0$ a.e. on Ω and (2) shows $u = \tilde{u}$. Thus part a) of the Theorem is proved. We note that for this step it is enough to have $g \in L^2(\Omega;\mathbb{R}^2)$ (compare (6)) which is also sufficient to carry out the next step.

Step 2: Higher weak differentiability of the minimizer u

We claim that

$$u \in W^2_{2,\mathrm{loc}}(\Omega;\mathbb{R}^2) \tag{10}$$

is true, which by Sobolev's theorem implies that u is locally Hölder continuous on Ω for any exponent $\alpha < 1$. For proving (10), we fix a coordinate direction i, $i = 1, 2$, and let Δ_h denote the difference quotient of functions in this direction. From the minimality of u we deduce

$$\int_\Omega \left(Df\left(\varepsilon^D(u)\right) : \varepsilon^D(v) - g\cdot v\right)\,dx = 0 \tag{11}$$

for all $v \in \overset{\circ}{W}{}^1_2(\Omega;\mathbb{R}^2)$. We let $v := \Delta_{-h}(\eta^2\Delta_h u)$ with $\eta \in C^\infty_0(\Omega)$, $0 \le \eta \le 1$, and deduce from (11)

$$\int_\Omega \Delta_h\left(Df\left(\varepsilon^D(u)\right)\right) : \varepsilon^D(\eta^2\Delta_h u)\,dx = -\int_\Omega g\cdot\Delta_{-h}(\eta^2\Delta_h u)\,dx\,. \tag{12}$$

Abbreviating

$$B_x := \int_\Omega^1 D^2 f\left(\varepsilon^D(u)(x) + th\,\varepsilon^D(\Delta_h u)(x)\right)\,dt$$

we get

$$\Delta_h\left(Df\left(\varepsilon^D(u)\right)\right) = B_x\left(\varepsilon^D(\Delta_h u)\,,\ \cdot\ \right)\,,$$

hence $\left(Z_h := \varepsilon^D(\eta^2\Delta_h u) - \eta^2\varepsilon^D(\Delta_h u)\right)$

$$\begin{aligned}
\text{l.h.s. of (12)} &= \int_\Omega B_x\left(\varepsilon^D(\Delta_h u), \varepsilon^D\left(\eta^2\Delta_h u\right)\right)\, dx \\
&= \int_\Omega B_x\left(\varepsilon^D(\Delta_h u), \varepsilon^D(\Delta_h u)\right)\eta^2\, dx + \int_\Omega B_x\left(\varepsilon^D(\Delta_h u), Z_h\right)\, dx \\
&\overset{(5)}{\geq} \lambda\int_\Omega \eta^2\left|\varepsilon^D(\Delta^h u)\right|^2\, dx - \Lambda\int_\Omega |\varepsilon^D(\Delta_h u)| 2\eta|\nabla\eta||\Delta_h u|\, dx \\
&\geq \frac{\lambda}{2}\int_\Omega \eta^2\left|\varepsilon^D(\Delta^h u)\right|^2\, dx - c\int_\Omega |\nabla\eta|^2\,|\Delta_h u|^2\, dx\,.
\end{aligned}$$

Therefore (12) implies

$$\frac{\lambda}{2}\int_\Omega \eta^2\left|\varepsilon^D(\Delta_h u)\right|^2\, dx \leq \int_\Omega |g||\Delta_{-h}\left(\eta^2\Delta_h u\right)|\, dx + c\int_\Omega |\nabla\eta|^2\,|\Delta_h u|^2\, dx\,. \tag{13}$$

If $|h| < \operatorname{dist}(\operatorname{spt}\eta, \partial\Omega)$, then we quote Lemma 7.23 of [GT] and get

$$\int_\Omega |\nabla\eta|^2\,|\Delta_h u|^2\, dx \leq \|\nabla\eta\|^2_{L^\infty(\Omega)}\int_\Omega |\nabla u|^2\, dx\,. \tag{14}$$

At the same time we have with Young's inequality

$$\begin{aligned}
\int_\Omega |g|\left|\Delta_{-h}(\eta^2\Delta_h u)\right|^2\, dx &\leq \delta\int_\Omega \left|\Delta_{-h}(\eta^2\Delta_h u)\right|^2\, dx + c(\delta)\int_\Omega |g|^2\, dx \\
&\leq \delta\int_\Omega \left|\nabla\left(\eta^2\Delta_h u\right)\right|^2\, dx + c(\delta)\int_\Omega |g|^2\, dx \\
&\overset{(2)}{\leq} 2\delta\int_\Omega \left|\varepsilon^D(\eta^2\Delta_h u)\right|^2\, dx + c(\delta)\int_\Omega |g|^2\, dx \\
&\leq 2\delta\int_\Omega \eta^2\left|\varepsilon^D(\Delta_h u)\right|^2\, dx \\
&\quad + c(\delta)\left[\int_\Omega |g|^2\, dx + \|\nabla\eta\|^2_{L^\infty(\Omega)}\int_\Omega |\nabla u|^2\, dx\right],
\end{aligned}$$

where we also used Lemma 7.23 of [GT] again. Combining (13) with (14) and the above estimate, we deduce after appropriate choice of δ:

$$\int_\Omega \left|\Delta_h\varepsilon^D(u)\right|^2\eta^2\, dx \leq c\left[\int_\Omega |g|^2\, dx + \|\nabla\eta\|^2_{L^\infty(\Omega)}\int_\Omega |\nabla u|^2\, dx\right]. \tag{15}$$

Since (15) is valid for all $|h| \ll 1$, we see that $\varepsilon^D(u)$ is of class $W^1_{2,\mathrm{loc}}(\Omega;\mathbb{S}^2)$. Therefore $\varepsilon^D(\partial_i u) = \partial_i \varepsilon^D(u)$ is in $L^2_{\mathrm{loc}}(\Omega;\mathbb{S}^2)$ and we infer from (3) that $\partial_i u \in W^1_{2,\mathrm{loc}}(\Omega;\mathbb{R}^2)$ is true for $i = 1, 2$. This proves our claim (10).

Step 3: $C^{1,\alpha}$–regularity of the minimizer u

Here we are going to prove part b) of the Theorem applying the hole filling technique going back to Widman's work [Wi]. Recall that according to our assumption (6) g is a bounded function but we remark that the following arguments can be carried out under the weaker hypothesis $g \in L^2(\Omega;\mathbb{R}^2)$ together with $g \in L^{2,\mu}_{\mathrm{loc}}(\Omega;\mathbb{R}^2)$ for some $\mu > 0$, where $L^{2,\mu}_{\mathrm{loc}}(\ldots)$ is the local Morrey space (see, e.g. [Gi], Chapter III). From (10) and (11) we get (from now on summation w.r.t. $k = 1, 2$)

$$\text{(16)} \qquad \int_\Omega \partial_k \left(Df\left(\varepsilon^D(u)\right)\right) : \varepsilon^D\left(\eta^2 \partial_k[u - Px]\right)\, dx = -\int_\Omega g \cdot \partial_k\left(\eta^2 \partial_k[u - Px]\right)\, dx$$

valid for $\eta \in C^\infty_0(\Omega)$, $0 \le \eta \le 1$, and any (2×2)–matrix P.
Letting $W_k := \varepsilon^D(\eta^2 \partial_k[u - Px]) - \eta^2 \varepsilon^D(\partial_k u)$ and observing $|W_k| \le c\eta|\nabla\eta||\nabla u - P|$, we deduce from (16) (recalling also (5))

$$\begin{aligned}
&\int_\Omega \eta^2 \left|\nabla \varepsilon^D(u)\right|^2\, dx \\
&\quad \le\; c\left[\int_\Omega \eta|\nabla\varepsilon^D(u)||\nabla\eta||\nabla u - P|\, dx + \int_\Omega |g||\nabla\eta|\eta|\nabla u - P|\, dx + \int_\Omega |g|\eta^2|\nabla^2 u|\, dx\right],
\end{aligned}$$

and if we apply Young's inequality we find

$$\text{(17)} \quad \int_\Omega \eta^2 \left|\nabla \varepsilon^D(u)\right|^2\, dx \le c\left[\int_\Omega |\nabla\eta|^2 \,|\nabla u - P|^2\, dx + \int_\Omega |g|^2\eta^2\, dx + \int_\Omega |g|\eta^2|\nabla^2 u|\, dx\right].$$

Let us fix a coordinate direction k. We have

$$\begin{aligned}
\int_\Omega \eta^2\, |\nabla\partial_k u|^2\, dx &= \int_\Omega \eta^2\, |\nabla(\partial_k u - P_k)|^2\, dx \\
&\le c\left[\int_\Omega \left|\nabla\left(\eta^2[\partial_k u - P_k]\right)\right|^2\, dx + \int_\Omega |\nabla\eta|^2\, |\nabla u - P|^2\, dx\right] \\
&\overset{(2)}{\le} c\left[\int_\Omega \left|\varepsilon^D\left(\eta^2[\partial_k u - P_k]\right)\right|^2\, dx + \int_\Omega |\nabla\eta|^2\, |\nabla u - P|^2\, dx\right]
\end{aligned}$$

$$\leq \; c \left[\int\limits_{\Omega} \eta^2 \left|\varepsilon^D(\partial_k u)\right|^2 \, dx + \int\limits_{\Omega} |\nabla \eta|^2 \, |\nabla u - P|^2 \, dx \right]$$

$$\leq \; c \left[\int\limits_{\Omega} \eta^2 \left|\nabla \varepsilon^D(u)\right|^2 \, dx + \int\limits_{\Omega} |\nabla \eta|^2 \, |\nabla u - P|^2 \, dx \right], \qquad \text{i.e.}$$

$$\int\limits_{\Omega} \eta^2 \left|\nabla^2 u\right|^2 \, dx \leq c \left[\int\limits_{\Omega} \eta^2 \left|\nabla \varepsilon^D(u)\right|^2 \, dx + \int\limits_{\Omega} |\nabla \eta|^2 \, |\nabla u - P|^2 \, dx \right], \tag{18}$$

and we may insert (17) into the r.h.s. of (18) with the result

$$\int\limits_{\Omega} \eta^2 \left|\nabla^2 u\right|^2 \, dx \leq c \left[\int\limits_{\Omega} |\nabla \eta|^2 \, |\nabla u - P|^2 \, dx + \int\limits_{\Omega} |g|^2 \eta^2 \, dx + \int\limits_{\Omega} |g| \eta^2 |\nabla^2 u| \, dx \right]. \tag{19}$$

If we use Young's inequality for the last integral in (19), it is shown that

$$\int\limits_{\Omega} \eta^2 \left|\nabla^2 u\right|^2 \, dx \leq c \left[\int\limits_{\Omega} \eta^2 \, |g|^2 \, dx + \int\limits_{\Omega} |\nabla \eta|^2 \, |\nabla u - P|^2 \, dx \right]. \tag{20}$$

In a final step we fix a disc $B_{2R}(x_0) \Subset \Omega$ and choose η such that $\eta = 1$ on $B_R(x_0)$, $\operatorname{spt} \eta \subset B_{2R}(x_0)$ and $|\nabla \eta| \leq c/R$. Inequality (20) implies

$$\int_{B_R(x_0)} \left|\nabla^2 u\right|^2 \, dx \leq c \left[R^2 + R^{-2} \int_{T_R(x_0)} |\nabla u - P|^2 \, dx \right], \tag{21}$$

where $T_R(x_0) := B_{2R}(x_0) - B_R(x_0)$. Choosing P as the mean value of ∇u over the ring $T_R(x_0)$ and applying Poincaré's inequality, we deduce from (21)

$$\int_{B_R(x_0)} \left|\nabla^2 u\right|^2 dx \leq c \left[R^2 + \int_{T_R(x_0)} \left|\nabla^2 u\right|^2 \, dx \right].$$

Adding $c \int_{B_R(x_0)} |\nabla^2 u|^2 \, dx$ on both sides we find

$$\int_{B_R(x_0)} \left|\nabla^2 u\right|^2 \, dx \leq \frac{c}{c+1} \, R^2 + \frac{c}{c+1} \int_{B_{2R}(x_0)} \left|\nabla^2 u\right|^2 \, dx \,,$$

and a standard iteration argument shows that $\int_{B_r(x_0)} |\nabla^2 u|^2 \, dx$ grows at most like r^κ for some $\kappa > 0$ which proves the claim of part b) of the Theorem.

□

REMARK 1. *1.) Having established the continuity of ∇u it is possible to prove the interior Hölder continuity of ∇u for any exponent $\alpha \in (0,1)$ by combining the freezing technique with Campanato type estimates valid for constant coefficient systems involving ε^D which have been established in [Sc].*

2.) Using these Campanato estimates one can also prove partial C^1–regularity for I–minimizers now on domains $\Omega \subset \mathbb{R}^n$, $n \geq 3$, and with iterative application of the difference quotient technique one obtains C^∞–regularity for minimizers of $\int\limits_\Omega [A(\varepsilon^D(u), \varepsilon^D(u)) - g \cdot u]\, dx$, $\Omega \subset \mathbb{R}^n, n \geq 2$, provided A is a positive definite, symmetric bilinear form on $\mathbb{S}^n$ (:= space of symmetric $(n \times n)$–matrices) having constant coefficients and g is a smooth function. For details we again refer to [Sc].

Let us look at the minimization problem

$$J[u,\Omega] := \int\limits_\Omega F(\varepsilon^D(u))\, dx \to \min \text{ in } u_0 + \overset{\circ}{W}{}^1_p(\Omega;\mathbb{R}^2) \tag{22}$$

with u_0 from the space $W^1_p(\Omega;\mathbb{R}^2)$ and with integrand $F : \mathbb{S}^2 \to \mathbb{R}$ being strictly convex satisfying in addition the growth condition

$$a|\sigma|^p - b \leq F(\sigma) \leq A|\sigma|^p + B,\ \sigma \in \mathbb{S}^2\,, \tag{23}$$

with constants $a, A > 0, b, B \geq 0$. The following Korn type inequality provides the existence of a unique solution to (22) under the hypothesis (23).

Proposition 1. *Let $\Omega \subset \mathbb{R}^2$ denote a bounded Lipschitz domain and let $p \in (1,\infty)$. Then there is a constant $C_p(\Omega)$ such that*

$$\|\nabla v\|_{L^p(\Omega)} \leq C_p(\Omega)\|\varepsilon^D(v)\|_{L^p(\Omega)} \tag{24}$$

holds for any $v \in \overset{\circ}{W}{}^1_p(\Omega;\mathbb{R}^2)$.

Proof: It is sufficient to consider the case $v \in C^\infty_0(\Omega;\mathbb{R}^2)$. From (26) in [Da] we deduce

$$\partial_i \varepsilon^D_{ij}(v) = \frac{1}{2}\Delta v^j, \quad j = 1,2\,,$$

i.e. $h := v^j \in \overset{\circ}{W}{}^1_p(\Omega)$ solves

$$-\Delta h = \operatorname{div} H \quad \text{on } \Omega \tag{25}$$

with $H := (-2\varepsilon^D_{ij}(v))_{1\leq i\leq 2}$ of course from the space $L^p(\Omega;\mathbb{R}^2)$. Applying standard L^p–arguments to equation (25) (see, e.g. [Me]) we end up with

$$\|\nabla h\|_{L^p(\Omega)} \leq C_p(\Omega)\|H\|_{L^p(\Omega)}$$

and our claim (24) follows.

□

If (23) is replaced by the stronger requirement that F is a smooth p–elliptic integrand, then we believe that the result of the Theorem extends to the p– case by combining the Korn type inequality (24) with suitable regularity techniques as applied for example in [BFZ].

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