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THRESHOLD STRESSES FOR DISLOCATION CLIMB OVER HARD PARTICLES: THE EFFECT OF AN ATTRACTIVE INTERACTION

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Abstract—We present a model for the effect of an attractive interaction between dislocations and hard spherical particles on the process of dislocation bypass by local climb. The interaction is treated by assigning a line tension to the dislocation which is lower in the vicinity of the particle than in the matrix. We find that even for very modest interactions, the strongest barrier to dislocation bypass is no longer provided by the climb obstacle, but rather by the detachment of the dislocation from the particle after climb over the particle is complete. The model provides a possible explanation for some experimentally observed dislocation configurations in crept ODS superalloys and for the creep thresholds which are typical of such alloys.

Résumé—Nous présentons un modèle pour l'influence d'une interaction attractive entre des dislocations et des particules sphériques dures sur le contournement par montée locale. Nous traitons l'interaction en assignant à le dislocation une tension de ligne qui est plus petite au voisinage de la particule que dans la matrice. Même pour des interactions très modestes, la plus forte barrière au franchissement de la particule par la dislocation n'est plus fournie par l'obstacle de montée, mais plutôt par le détachement de la dislocation et de la particule lorsque la montée est terminée. Le modèle fournit une explication possible pour quelques configurations de dislocations que l'on a observées expérimentalement dans des superalliages après fluage et pour les contraintes de fluage typiques de tels alliages.

Zusammenfassung—Das lokale Klettern von Versetzungen über harte kugelförmige Teilchen wird für den Fall einer anziehenden Wechselwirkung zwischen Versetzung und Teilchen modellmäßig beschrieben. Die Wechselwirkung wird berücksichtigt, indem der Versetzung in der Nähe des Teilchens eine erniedrigte Linienspannung zugeschrieben wird. Es zeigt sich, daß schon bei geringfügiger Wechselwirkung das entscheidende Ereignis nicht mehr der Kletterschritt, sondern das Ablösen der Versetzung vom Teilchen nach Beendigung des Überkletterns ist. Das Modell gibt eine mögliche Erklärung für experimentell beobachtete Versetzungskonfigurationen in kriechbeanspruchten ODS-Superlegierungen und für bei solchen Legierungen typische Kriechschwellspannungen.

1. INTRODUCTION

The creep rates of dispersion strengthened alloys are commonly found to be highly stress-sensitive, which suggests the existence of a threshold below which the creep rates become negligible. Available experimental data, e.g. [1–5], indicate that these threshold stresses are generally about half the Orowan stress, which is given in shear, by the following approximation

$$\tau_{or} = 0.84 \frac{Gb}{2(\lambda - r)} \tag{1}$$

where G is the shear modulus of the material, b the Burgers vector of a lattice dislocation, r the average radius of the particles and 2λ their mean planar spacing. There is disagreement in the literature over the exact form of equation (1), but at least two studies [1, 3] have shown conclusively, by normalization with respect to room temperature strength, that the creep threshold lies below the Orowan stress.

Explanations for this threshold must therefore invoke the possibility that, at creep temperatures, dislocations can "avoid" hard particles by climbing around them. This process has been modelled by Brown and Ham [6] and by Shewfelt and Brown [7], assuming that dislocation climb is localized at the particles. This means that only the portion of the dislocation which, although still a lattice dislocation, remains in close proximity with the particle–matrix interface undergoes climb; the portion of the dislocation not in contact with the particle remains in the glide plane. This is illustrated in Fig. 1. Since the dislocation line length increases as the dislocation climbs, extra energy must be supplied by the applied stress: this is the origin of the threshold stress. A computer simulation of this process in a random array of hard particles of low volume fraction by Shewfelt and Brown [7] has resulted in a relationship for the threshold stress given by

$$\tau_0^{loc} = 0.32 \frac{Gb}{2\lambda} \tag{2}$$

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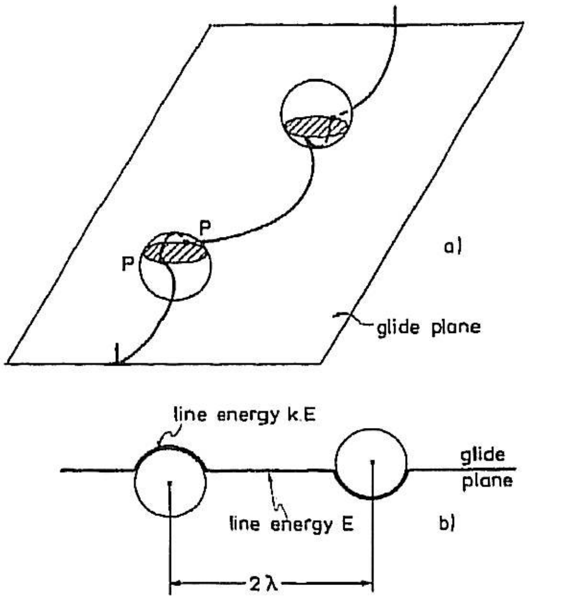

Fig. 1. The mechanism of local climb of a lattice dislocation over and under hard spherical particles of spacing 2λ: (a) perspective view, (b) side view.

which is in reasonable agreement with the experimental data. Arzt and Ashby [8] subsequently arrived at a similar result using a simple analytical model.

The primary criticism levelled at these models has involved the assumption of local climb. Lagneborg [9] has argued that the sharp dislocation curvatures at the point of particle contact (marked "P" in Fig. 1), which are necessary for local climb to occur, cannot be sustained. Rather a vacancy flux is generated which leads to more extensive ("general") climb of the dislocation. Lagneborg's model for this process leads to a back stress which however scales as the applied stress, and does not therefore predict a true threshold. It is now recognized that a small, but finite threshold does exist, even for general climb. It arises because in order to pass through a random array of particles a dislocation has to adopt a certain minimum curvature [4, 7, 8, 10, 11]. For volume fractions of less than about five percent, this threshold is approximately an order of magnitude lower than that for local climb and therefore well below the bulk of the experimental data.

Hence we need to explain why, even in tests conducted at very low strain rates, the lower threshold is not observed. One potential explanation is related to the details of the dislocation–particle interaction at high temperatures. Srolovitz *et al.* [12, 13] have recently modelled the effect of a particle–matrix interface which is unable to sustain shear tractions, on a dislocation. They show that an attractive interaction can result as the dislocation at the interface relaxes part of its strain field. In addition, TEM work on crept oxide-dispersion strengthened superalloys [14] and, in particular, more recent weak-beam studies [15] have revealed dislocation–particle configurations which suggest indeed that such an attractive interaction may exist. However, the cores of the dislocations residing near the interface still produce good contrast, suggesting that they have not fully relaxed. What is apparent from the micrographs is that the dislocations remain bound to the particles over which they have just climbed. Thus the detachment process may prove to be a stronger barrier to dislocation bypass than the climb barrier itself, and can provide an additional mechanism for a threshold stress.

In this paper we consider the effect that such an attractive interaction would have on the climb behaviour of a dislocation. In particular we have calculated the stress necessary to maintain the dislocation in equilibrium at any given point in the climb process, including the point of detachment from the particle. The maximum value of this stress is the threshold stress for creep.

2. THE MODEL

We consider the passage of a dislocation through a uniform array of spherical hard particles of radius r and spacing 2λ. The dislocation is assumed to overcome the obstacles by means of local climb. The non-climbing portion of the dislocation has a line energy per unit length E, which is given to a reasonable approximation by

$$E = \frac{Gb^2}{2}. \tag{3}$$

The attractive interaction between particle and dislocation is modelled by assuming that the line energy of the dislocation is relaxed at the particle interface to a value given by $E' = k \cdot E$. The parameter k can be thought of as a relaxation parameter, and takes on values between 0 and 1. When $k = 1$, no attractive interaction with the particle occurs. For $k = 0$, a maximum attraction results, with the interface behaving as if it were the surface of a void. This approach neglects, of course, any long-range interactions, which are much more difficult to handle.

The calculation is performed on a segment of dislocation of length 2λ, centered on a particle. The glide plane is assumed to lie a distance h above the equator of the particle. The geometry is shown in Fig. 2. The trajectory of the climb segment is given at any point by the great circle which joins the two points of particle contact (this being the shortest trajectory over a spherical surface). The non-contacting portion maintains an equilibrium configuration, in which the radius of curvature is

$$R = \frac{Gb}{2\tau} \tag{4}$$

τ being the applied shear stress.

We first calculate the energy U of the dislocation segment at any position

$$U = s_p \cdot E' + s_n \cdot E. \tag{5}$$

Here the arc length in contact with the particle is

$$s_p = 2r \cdot \sin^{-1}\left(\frac{a}{r}\right) \tag{6}$$

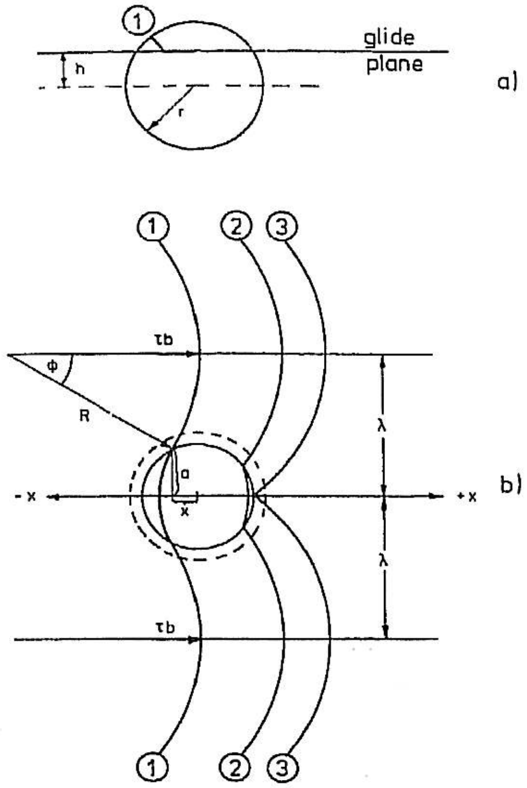

Fig. 2. The geometry of particle–dislocation interaction during local climb: (a) side view, (b) plan view. The dislocation, moving from left to right, is shown in three positions in (b): at "1" it is still on the arrival side, "2" is a typical configuration on the departure side (as it is also observed experimentally [15]), "3" is at the point of detachment.

where

$$a = \sqrt{r^2 - h^2 - x^2} \tag{7}$$

is the distance between the two points at which the dislocation contacts the particle. The arc length not in contact with the particle is

$$s_n = 2R \cdot \sin^{-1}\left(\frac{\lambda - a}{R}\right). \tag{8}$$

In order to determine the stress resisting the progress of the dislocation at any given position we allow the dislocation segment away from the particle to move forward by a virtual displacement dP. In so doing the points of contact move in the glide direction by dx. We then equate the work done by the dislocation dW with the change in energy of the dislocation segment dU. The former is simply

$$\mathrm{d}W = 2\tau b(\lambda - a)\,\mathrm{d}P. \tag{9}$$

The change of energy of the dislocation segment is

$$\mathrm{d}U = \frac{2rE'}{\sqrt{1 - \left(\frac{a}{r}\right)^2}}\frac{\mathrm{d}a}{r} + 2RE \cdot \mathrm{d}\phi \tag{10}$$

where

$$\mathrm{d}\phi = -\frac{\tau/\tau_n}{\sqrt{1 - (\tau/\tau_n)^2}}\frac{\mathrm{d}a}{\lambda - a}. \tag{11}$$

Here τ_n is a normalizing stress equal to

$$\tau_n = \frac{Gb}{2(\lambda - a)}. \tag{12}$$

The two displacements are related by simple geometry, such that

$$\mathrm{d}P = \mathrm{d}x + R \cdot \sin\phi \cdot \mathrm{d}\phi. \tag{13}$$

By equating dU and dW, and after some manipulation, one finds the following implicit equation for the resistance τ

$$\frac{\tau}{\tau_n} + \frac{x}{a}\frac{k}{\sqrt{1 - \left(\frac{a}{r}\right)^2}} = \frac{x}{a}\sqrt{1 - \left(\frac{\tau}{\tau_n}\right)^2}. \tag{14}$$

This can be readily solved to yield two possible solutions

$$\frac{\tau}{\tau_0} = \frac{\frac{\lambda}{r} - 1}{\frac{\lambda}{r} - \frac{a}{r}}\frac{\frac{x}{a}}{1 + \left(\frac{x}{a}\right)^2}\left\{-\frac{k}{\sqrt{1 - \left(\frac{a}{r}\right)^2}} \pm \sqrt{1 + \left(\frac{x}{a}\right)^2\left[1 - \frac{k^2}{1 - \left(\frac{a}{r}\right)^2}\right]}\right\} \tag{15}$$

where τ_0 is the "nominal" Orowan stress, i.e.

$$\tau_0 = \frac{Gb}{2(\lambda - r)}. \tag{16}$$

Further examination of equation (15) reveals that only the solution containing the + sign is physically significant.

3. DISCUSSION

Equation (15) determines the stress necessary for local dislocation climb to continue at any given point in the particle/matrix interface. Two limiting cases are worth noting. First when $x = 0$, at the pole of the particle, the stress is also zero, as must be the case; in fact the solution is antisymmetric with respect to the origin. Second, at the point of detachment of the dislocation from the particle ($a = 0$, $x > 0$) the stress is given by

$$\tau_d = \frac{Gb}{2\lambda}\sqrt{1 - k^2}. \tag{17}$$

This is the same result as derived previously by Russell and Brown [17] to describe the strength imparted by shearable particles whose elastic modulus is lower by a factor k than that of the matrix. (Their additional expression for weak obstacles gives only slight deviations from this solution for k close to one where, as will be shown below, the detachment threshold is usually not controlling. We therefore use the strong obstacle approximation throughout this paper.) This detachment stress is determined by the energy balance at the point where the length of dislocation line in contact with the particle shrinks to

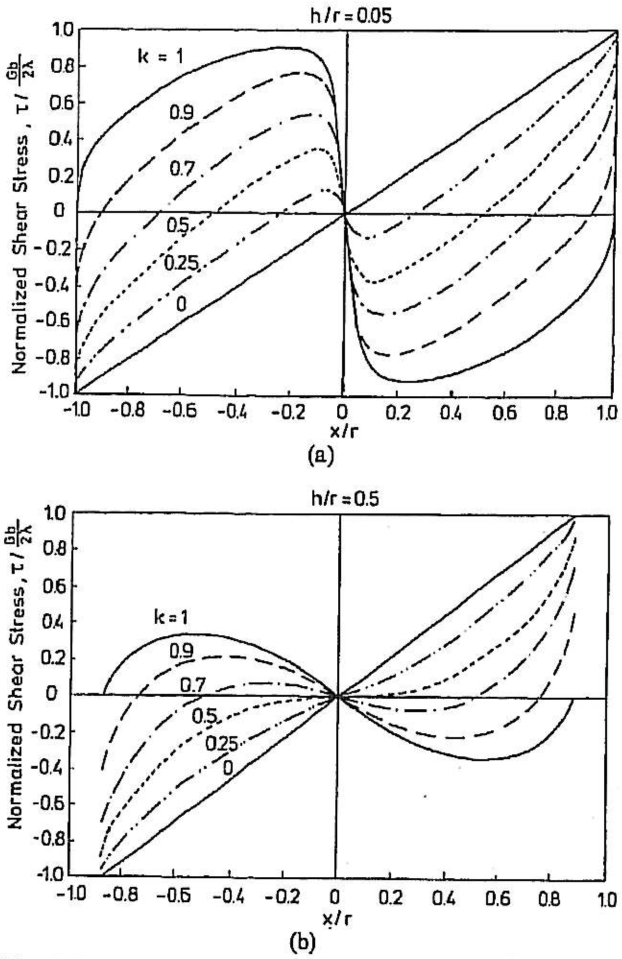

Fig. 3. The normalized shear stress vs dislocation position for different values of the relaxation parameter k, in the limiting case of zero volume fraction: (a) $h/r = 0.05$, (b) $h/r = 0.5$.

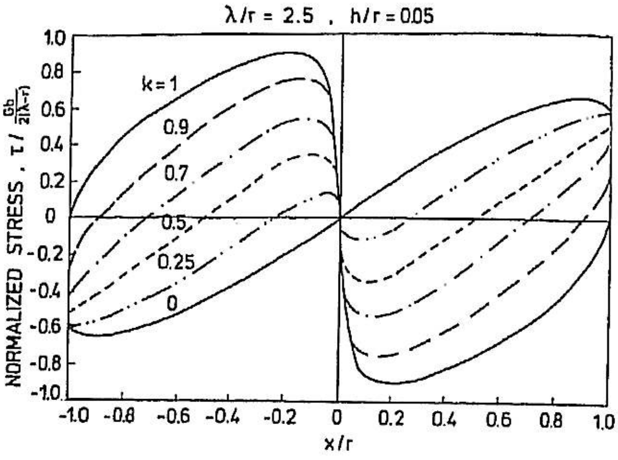

Fig. 4. The normalized shear stress vs dislocation position for different values of k at low volume fraction $f_v = 8.4\%$ ($\lambda/r = 2.5$) and $h/r = 0.05$.

zero. The result is therefore independent of the process by which the dislocation moves over the particle, whether it be local or general climb. It is also independent of particle size.

In general, the stress resisting dislocation climb does depend on the particle volume fraction f_v, through the parameter λ/r. The geometric parameters x and a enter only as quantities normalized with respect to the particle radius. Particle size as such is therefore irrelevant for the creep threshold as long as it is small compared to the particle spacing. This of course applies only to the energetics of particle climb considered here, and not to the kinetics of the process, as will be discussed elsewhere [16].

For reasons that will become clear, the results are most easily understood by first considering the case of low volume fraction, corresponding to $\lambda/r > 2$, i.e. $f_v < 13\%$. This limit includes all the data on creep threshold stresses in the literature for oxide dispersion-strengthened systems.

3.1. *Low volume fraction, $f_v < 13\%$ ($\lambda/r > 2$)*

In the limit as the volume fraction approaches zero (i.e. $r \ll \lambda$), the first term in equation (15) approximates to one. The results for this limit are shown in Fig. 3, in which the stress is plotted as a function of dislocation position x/r, for a range of values of the relaxation parameter k, and two values of h/r. When $k = 1$, the case of no attractive interaction, the stress resisting the progress of the dislocation rises as the dislocation climbs to the top of the particle, then becomes negative on the departure side. As k decreases, the stresses on the arrival side decrease, while those on the departure side increase. This reflects the effect of the changing energy balance as matrix dislocation is exchanged for climbing dislocation of different arc length and line tension. If k is low enough the stress is entirely negative on the arrival side. This means that the dislocation is pulled up to the top of the particle, and must then do work on the departure side to leave the particle.

Figure 3 shows that the maximum stress occurs either on the arrival side during climb or at the point of departure, depending on the values of k and h/r. The former leads to a climb barrier for dislocation bypass, while the latter results in a detachment barrier.

Figure 4 shows the stress as given by equation (15) for a small but finite volume fraction ($f_v = 8.4\%$) corresponding to $\lambda/r = 2.5$. While the shapes of the curves are somewhat different from the zero volume fraction limit, the general features are unaltered. It is clear that as the volume fraction of second phase particles increases the size of the climb barrier increases relative to the detachment barrier. This results because the climb barrier scales roughly with $1/(\lambda - r)$ while the detachment stress depends only on $1/\lambda$.

Whereas the detachment stress, given by equation (17), does not depend on the height-of-intersection of the glide plane at the particle, the maximum climb resistance is a function of h/r, as shown in Fig. 5. In a real particle dispersion, a given glide plane intersects the individual particles at random values of h/r. In order to determine the average value of the climb threshold, the following approximate relationship for τ_c as a function of k and h/r was generated by

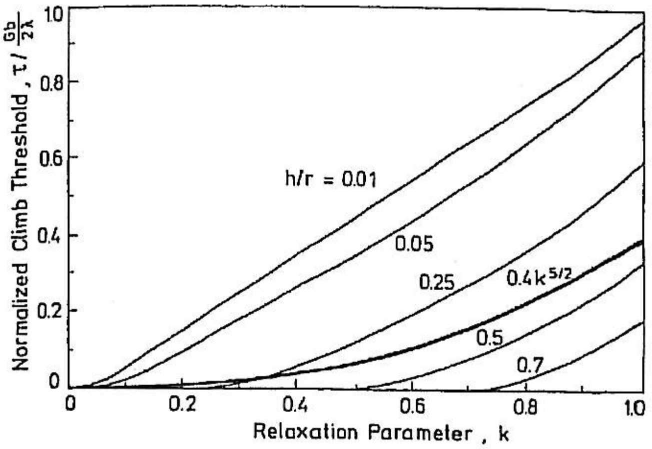

Fig. 5. The normalized maximum climb stress vs the relaxation parameter k for different values of h/r (thin lines) and for an array of particles which are randomly intersected by the glide plane (thick line).

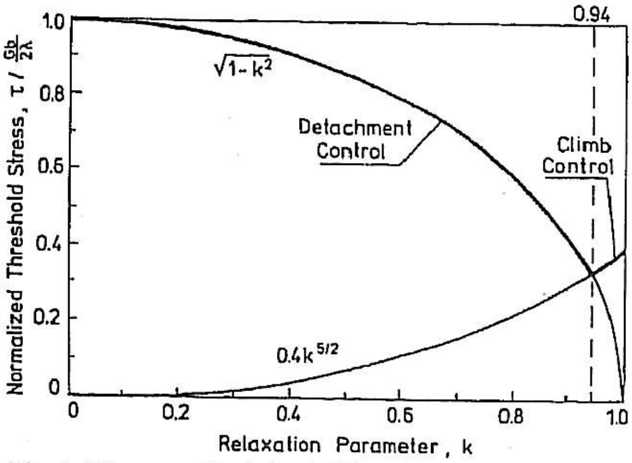

Fig. 6. The normalized threshold stresses for climb and for detachment vs the relaxation parameter k, case of zero volume fraction. When $k < 0.94$, i.e. a very modest interaction, the detachment process determines the threshold stress.

numerical fitting

$$\tau_c = \tau_0\left(k - \frac{h}{r}\right)^n \quad \text{when} \quad \frac{h}{r} < k$$
$$\tau_c = 0 \quad \text{when} \quad \frac{h}{r} \geq k \tag{18}$$

where $n = 3/2$. This fit results in maximum errors for τ_c/τ_0 of ± 0.1. By assuming that the glide planes are equally distributed over all values of h/r, the average climb threshold is obtained by integration of equation (18), to give

$$(\tau_c)_{ave} = 0.4k^{5/2}\tau_0. \tag{19}$$

This is also plotted as a function of k in Fig. 5. For $k = 1$, the average climb resistance is equal to $0.4\tau_0$, which is very similar to the results obtained by Shewfelt and Brown [7] and by Arzt and Ashby [8]. It should be noted, however, that this result is obtained assuming a uniform particle distribution on the glide plane whereas Shewfelt and Brown simulated the case of a random distribution. It would therefore appear that the factor of 0.4 is due to the statistics of h/r only and is not sensitive to assumptions about the distribution of the particles in the glide plane.

The overall threshold stress for dislocation bypass is simply the largest value of the resistance stresses given by equations (17) and (19). The results are plotted in Fig. 6 for the limit of zero volume fraction. The critical value of k below which dislocation bypass becomes detachment controlled is 0.94. It depends only weakly on λ/r, and falls to 0.85 at $f_v = 13\%$ ($\lambda/r = 2$). Therefore only a rather weak attractive interaction is required before the barrier governing the threshold stress changes from climb on the arrival side of the particle to the detachment process on the departure side. The weak dependence of the critical value of k, k_{crit}, on r/λ can be fit to a good approximation by

$$k_{crit} = 0.94\left(1 - \frac{r}{\lambda}\right)^{0.073}. \tag{20}$$

The transition would be shifted to even higher k-values if the climb process was "general" instead of "local".

The model predicts that dislocations trapped at attractive particles are most likely to be found on the departure side where the resistance is greatest, in a configuration in which the dislocations are bowed away from the particles [Fig. 2(b), dislocation "2"]. Such configurations have indeed been observed in ODS superalloys [14, 15]. Using the model, it is possible in principle to estimate the actual value of the relaxation parameter k [16].

3.2. Large volume fractions, $f_v > 13\%$ ($\lambda/r < 2$)

Figure 7 shows the stress resisting climb for $\lambda/r = 1.1$ (corresponding to a volume fraction of 0.43). For such a large volume fraction the nature of the climb process is quite different. The length of the extra dislocation line which must be produced during climb is proportionately much greater than at low volume fraction. As a result, the detachment stress is much less important. Moreover the maximum climb

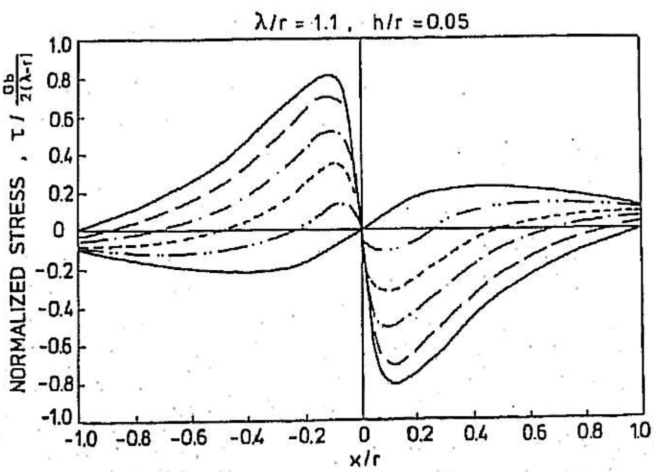

Fig. 7. The normalized shear stress vs dislocation position for different values of k at high volume fraction $f_v = 43\%$ ($\lambda/r = 1.1$) and $h/r = 0.05$.

resistance may be found either on the arrival or the departure side of the particle. We find also that once λ/r is less than 2, the approximate equations derived for the climb stress given by equations (18) and (19) are no longer valid. The model itself is also of little practical significance since attractive interactions of the type postulated here will only be produced at incoherent interfaces, for example in dispersion strengthened materials in which volume fractions are typically less than 10%.

4. CONCLUSIONS

1. The effect of an attractive interaction between a hard particle and a lattice dislocation climbing "locally" around it can be modelled to give an analytic expression which determines the stress necessary for climb to continue. At small volume fractions (< 13%) corresponding to $\lambda/r > 2$, the threshold stress for climb over particles which are randomly intersected by the glide plane is given (to a good approximation) by

$$(\tau_c)_{ave} = 0.4k^{5/2}\tau_0$$

where τ_0 is the "nominal" Orowan stress and k the factor by which the dislocation line tension is lowered in the vicinity of the particle.

2. The attractive interaction causes an additional threshold stress which must be exceeded in order to detach the dislocation from the particle after climb is completed

$$\tau_d = \frac{Gb}{2\lambda}\sqrt{1-k^2}.$$

This threshold applies irrespective of whether the preceding climb process was "local" or "general".

3. Unlike the climb threshold, which scales as the Orowan stress and therefore as $1/(\lambda - r)$, the detachment threshold depends only on $1/\lambda$ and not on particle size.

4. Only a very modest attractive interaction between dislocations and particles is required in order for dislocation detachment to become the strength-controlling process. This conclusion is supported by TEM observations of dislocation configurations around particles in ODS superalloys. One might also expect that other dispersion-strengthened alloys derive their high temperature strength from a resistance to dislocation detachment.

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